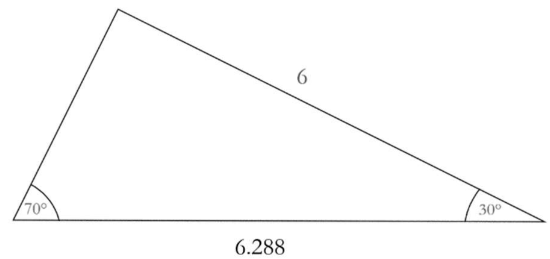
The Sine Area Rule

$$A = \frac{1}{2}ab\sin C$$

This equation can be used to find the area of a triangle (represented by A) when its height is unknown, using the function $\sin C$. $\sin C$ means to carry out the function of $\sin$ on the value of the angle C. "a" and "b" represent the lengths of two sides and "C" is the angle between those sides.

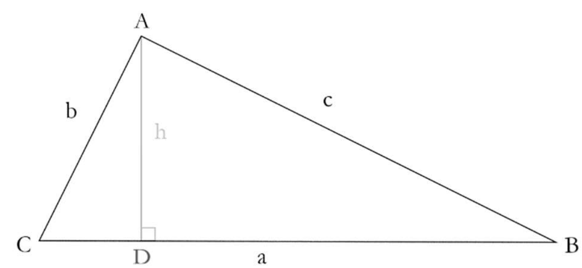
For example, to find the area of the triangle to the right, we would use $a = 6, b = 7$ (although $a = 7, b = 6$ would work equally well) and $C = 30°$ as this angle is sandwiched between those two sides, rather than $70°$. So the area of the triangle can be found:



$$A = \frac{1}{2} \times 6 \times 6.288 \times \sin 30°$$
$$A = \frac{1}{2} \times 6 \times 6.288 \times 0.5$$
$$A = 9.432$$

Proof

Consider the triangle to the right. Typically, the area of a triangle can be found by the equation $A = \frac{1}{2} \times a \times h$ where a is the base length of the triangle and h is the height of the triangle.



So, using the basic formula for the area of a triangle

$$A = \frac{1}{2} \times a \times h$$

Using SOH CAH TOA

$$\sin\theta = \frac{opposite}{hypotenuse}$$

As ΔACD is clearly right-angled

$$\sin C = \frac{h}{b}$$

Re-arranging gives

$$h = b\sin C$$

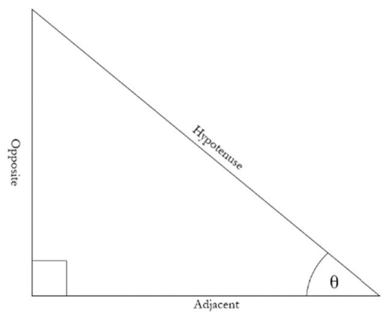
Substituting this value into the original formula

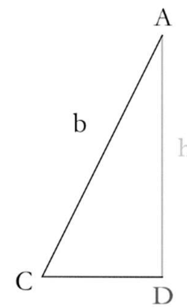
$$A = \frac{1}{2} \times a \times b\sin C$$

Therefore

$$A = \frac{1}{2}ab\sin C$$

For reference, here is the general form for a right-angled triangle alongside the triangle ACD used to find $\sin \theta$ in step 2 and $\sin C$ in step 3.





<u>See also</u>
- SOH CAH TOA
- Sine Rule
- Cosine Rule

<u>References</u>
Attwood, G. et al. (2017). *Edexcel AS and A level Mathematics - Pure - Year 1*. London: Pearson Education. p.185